UPR-658-T

Four-dimensional supersymmetric dyonic black holes in eleven-dimensional supergravity

Mirjam Cvetič * and Donam Youm †
Physics Department
University of Pennsylvania, Philadelphia PA 19104-6396
(May 1995)

Abstract

A class of 4-dimensional supersymmetric dyonic black hole solutions that arise in an effective 11-dimensional supergravity compactified on a 7-torus is presented. We give the explicit form of dyonic solutions with diagonal internal metric, associated with the Kaluza-Klein sector as well as the three-form field, and relate them to a class of solutions with a general internal metric by imposing a subset of $SO(7) \subset E_7$ transformations. We also give the field transformations which relate the above configurations to 4-dimensional ground state configurations of Ramond-Ramond and Neveu-Schwarz-Neveu-Schwarz sector of type-IIA strings on a 6-torus.

I. INTRODUCTION

In theories that attempt to unify gravity with other forces of nature, in particular, effective theories from superstrings, the non-trivial configurations, *i.e.*, topological defects as well as black holes (BH's), provide an important testing ground to address the role of gravity in such theories. Configurations which saturate the Bogomol'nyi bound on their energy (ADM mass) correspond to the ground state configurations within its class. Supersymmetric embedding of such configurations ensures that they are invariant under (constrained) supersymmetry transformations, *i.e.*, they satisfy the corresponding Killing spinor equations. For the above reasons, one refers to such configurations as supersymmetric, and in the case of spherically symmetric configurations, as Bogomol'nyi-Prasad- Sommerfeld (BPS) saturated states.

BPS saturated states within effective (super)gravity theories are of special interest, since they shed light on the nature of non-trivial ground states in such theories. In view of

*E-mail address: cvetic@cvetic.hep.upenn.edu

†E-mail address: youm@cvetic.hep.upenn.edu

recently conjectured duality [1–3] between strongly coupled 10-d type-IIA superstring theory and 11-d supergravity (SG) theory compactified on a circle as well as related dualities in other dimensions [4,5] [1], *e.g.*, the strong-weak coupling limits of type-IIA string theory compactified on K_3 surface, and the 11-d SG and heterotic string theory compactified on a torus, it is important to have a systematic and explicit construction of *all* the supersymmetric configurations of the corresponding effective SG theories in different dimensions. Such a systematic study would, in turn, provide a ground work for testing explicitly the conjectured duality symmetries between the corresponding effective (weakly coupled) SG theories and the corresponding strongly coupled string theories in different dimensions, at least at the level of the light (ground state) spectrum of states. The proposed construction of all the BPS saturated states is a monumental task, however, some of its aspects have been addressed for certain types of supersymmetric configurations in certain dimensions [4,5,8,9].

In this paper, we would like to single out a specific aspect of the proposed study. Namely, we would like to set up an explicit construction of 4-d BPS saturated BH solutions of 11-d SG compactified on a 7-torus [2]. These configurations would, in turn, allow for the testing of the conjectured duality between these states and the light spectrum of the corresponding strongly coupled string theory(ies) in 4-d.

BPS saturated states, corresponding to four-dimensional (4-d), spherically symmetric, static BH solutions, have masses which are related to the electric and magnetic charges of the BH. With the electric and magnetic charges quantized, masses of such BH's are related to the multiples of the elementary electric and magnetic charges. A subset of 4-d supersymmetric BH's within different sectors of effective 4-d SG theories has been already addressed [3]. However, results were primarily obtained in special cases with either non-zero electric or non-zero magnetic charges. In addition, only a subset of scalar fields was turned on [4].

Recently, progress has been made in finding the explicit form of all the 4-d supersymmetric as well as all the non-extreme BH's in Abelian $(4+n)$-d Kaluza-Klein (KK) theory. Those are BH's with the $U(1)$ gauge fields and scalar fields which originate from a $(4+n)$-d metric turned on. Supersymmetric BH's consist of n electric charges $\vec{\mathcal{Q}} \equiv (Q_1, ..., Q_n)$ and n magnetic charges $\vec{\mathcal{P}} \equiv (P_1, ..., P_n)$ which are subject to the constraint $\vec{\mathcal{P}} \cdot \vec{\mathcal{Q}} = 0$ [23]. The generating solutions are supersymmetric $U(1)_M \times U(1)_E$ BH's [24], *i.e.*, dyonic BH's with

---

[1] For a related recent works see Refs. [6–12] and references therein.

[2] Compactification on a 7-torus, *i.e.*, the internal isometry group is Abelian, provides the simplest possible compactification, and thus the first one to be addressed.

[3] See Refs. [13–22] and references therein.

[4] Recently, a general class of electrically charged, rotating BH solutions in the heterotic string theory compactified on a 6-torus has been constructed [20], and a procedure to construct the corresponding solutions with a general electric and magnetic charge configurations has been spelled out. Supersymmetric limit of the latter configurations, however, has not been addressed, yet.

one electric and one magnetic charges arising from different $U(1)$ factors, which correspond to supersymmetric BH's with a diagonal internal metric Ansatz. All the supersymmetric BH's are obtained by performing the $SO(n)/SO(n-2)$ rotations, which do not affect the 4-d space-time metric and the volume of the internal space, on the generating solution, *i.e.*, the supersymmetric $U(1)_M \times U(1)_E$ solution. The explicit form for all the 4-d, Abelian, static, spherically KK BH's is obtained [25] by performing two $SO(1,1)$ boosts on the non-extreme $U(1)_M \times U(1)_E$ KK BH and supplementing it by $SO(n)/SO(n-2)$ transformations.

The aim of this paper is a few fold. First, we provide an explicit construction of 4-d supersymmetric BH configurations which arise from two different sectors of 11-d SG theory compactified on a 7-torus. The first one is the configurations whose charges arise from $U(1)$ gauge fields associated with the 11-d metric (KK BH's). Abelian BH's in the $(4+n)$-d KK theory with $n=7$ constitute this subset of BH solutions. The second class of configurations are those whose charges arise from the compactification of the three-form field $A^{(11)}_{MNP}$ of 11-d SG. The explicit general construction of the latter configurations, with the scalar fields of the internal metric and the volume of the internal space turned on, constitutes a major part of this paper. In addition, we address the symmetry structure of such configurations and the procedure to obtain an explicit form of all such ground state configurations. The two classes of such solutions provide the initial building blocks to construct all the supersymmetric BH's of 11-d SG on a 7-torus.

Another aim is to address the connection between the strongly coupled states of type-IIA string theory and those of weakly coupled 11-d SG. In particular, we find the explicit field transformations between 4-d solutions of 11-d SG and Ramond-Ramond (R-R) as well as Neveu-Schwarz- Neveu-Schwarz (NS-NS) sector of type-IIA superstring theory compactified on a 6-torus.

The paper is organized as follows. In chapter II, we summarize the properties of 11-d SG theory and its compactification down to 4-d. In chapter III, we study two classes of Abelian charged BH solutions, each of which are associated with KK gauge fields and 3-form $U(1)$ gauge fields, respectively. In chapter III, we discuss the connection between the 11-d SG and type-IIA, as well as heterotic superstring theory, and comment on the strong-weak coupling behavior among such theories. In chapter IV, conclusions are given.

II. ELEVEN-DIMENSIONAL SUPERGRAVITY AND ITS COMPACTIFICATION DOWN TO FOUR-DIMENSIONS

In this section, we summarize the particle content and the effective Lagrangian density of 11-d supergravity (SG) compactified down to 4-d on a 7-torus. The field content of the N=1, d=11 SG is the Elfbein $E^{(11)\,A}_M$, gravitino $\psi^{(11)}_M$, and the 3-form field $A^{(11)}_{MNP}$. The bosonic Lagrangian density is given by [26]

$$\mathcal{L} = -\frac{1}{4}E^{(11)}[\mathcal{R}^{(11)} + \frac{1}{12}F^{(11)}_{MNPQ}F^{(11)\ MNPQ} - \frac{8}{12^4}\varepsilon^{M_1\cdots M_{11}}F_{M_1\cdots M_4}F_{M_5\cdots M_8}A_{M_9M_{10}M_{11}}], \quad (1)$$

where $E^{(11)} \equiv \det E^{(11)\ A}_M$, $\mathcal{R}^{(11)}$ is the Ricci scalar defined in terms of the Elfbein, and $F^{(11)}_{MNPQ} \equiv 4\partial_{[M}A^{(11)}_{NPQ]}$ is the field strength associated with the 3-form field $A^{(11)}_{MNP}$. The

supersymmetry transformation of the gravitino field $\psi_M^{(11)}$ in the bosonic background is given by

$$\delta\psi_M^{(11)} = D_M\,\varepsilon + \frac{i}{144}(\Gamma^{NPQR}{}_M - 8\Gamma^{PQR}\delta_M^N)F_{NPQR}\,\varepsilon, \tag{2}$$

where $D_M\,\varepsilon = (\partial_M + \frac{1}{4}\Omega_{MAB}\Gamma^{AB})\,\varepsilon$ is the gravitational covariant derivative on the spinor ε, and $\Omega_{ABC} \equiv -\tilde{\Omega}_{AB,C} + \tilde{\Omega}_{BC,A} - \tilde{\Omega}_{CA,B}$ ($\tilde{\Omega}_{AB,C} \equiv E^{(11)\,M}_{[A}E^{(11)\,N}_{B]}\partial_N E^{(11)}_{MC}$) is the spin connection defined in terms of the Elfbein. Our convention for the metric signature is $(+--\cdots-)$. For the space-time vector index convention, the characters $(A, B, ...)$ and $(M, N, ...)$ denote flat and curved indices, respectively.

The dimensional reduction of 11-d theory down to 4-d is achieved by taking the KK Ansatz for the Elfbein and a consistent truncation of the other 11-d fields [27]. With the internal space being a 7-torus T^7 ,*i.e.*, the internal isometry group is Abelian, all the fields are independent of the internal coordinates in the zero mode approximation. One can use the off-diagonal part of local Lorentz $SO(1,10)$ invariance to put the Elfbein into the following triangular form:

$$E_M^{(11)\,A} = \begin{pmatrix} e^{-\frac{\varphi}{2}}e^\alpha_\mu & B^i_\mu e^a_i \\ 0 & e^a_i \end{pmatrix}, \tag{3}$$

where $\varphi \equiv \ln\det e^a_i$, and B^i_μ $(i = 1, ..., 7)$ are KK Abelian gauge fields. Here, we use the greek letters $(\alpha, \beta, \cdots)$ $[(\mu, \nu, \cdots)]$ for the 4-d space-time flat [curved] indices and the latin letters $(a, b, \cdots)$ $[(i, j, \cdots)]$ for the internal flat [curved] space indices. The 3-form field $A^{(11)}_{MNP}$ is truncated into the following three types of 4-d fields: 35 pseudo-scalar fields A_{ijk}, 21 pseudo-vector fields $A_{\mu\,ij}$ and 7 two-form fields $A_{\mu\nu\,i}$. The two-form fields $A_{\mu\nu\,i}$ are equivalent to (axionic) scalar fields φ^i after making duality transformation. In order to ensure that the fields $A_{\mu\,ij}$ and $A_{\mu\nu\,i}$ are scalars under the internal coordinate transformation $x^i \to x'^{\,i} = x^i + \xi^i$, and transform as $U(1)$ gauge fields under the gauge transformation $\delta A^{(11)}_{MNP} = \partial_M\zeta_{NP} + \partial_N\zeta_{PM} + \partial_P\zeta_{MN}$, one has to define new canonical 4-d fields in the following way:

$$A'_{\mu\,ij} \equiv A_{\mu\,ij} - B^k_\mu A_{kij}, \qquad A'_{\mu\nu\,i} \equiv A_{\mu\nu\,i} - B^j_\mu A_{j\nu\,i} - B^j_\nu A_{\mu\,ji} + B^j_\mu B^k_\nu A_{jki}. \tag{4}$$

Then, the bosonic action (1) reduces to the following effective 4-d action:

$$\mathcal{L} = -\frac{1}{4}e[\mathcal{R} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{1}{4}\partial_\mu g_{ij}\partial^\mu g^{ij} - \frac{1}{4}e^\varphi g_{ij}G^i_{\mu\nu}G^{j\,\mu\nu} + \frac{1}{2}e^\varphi g^{ik}g^{jl}F^4_{\mu\nu\,ij}F^{4\,\mu\nu}{}_{kl} + \cdots], \tag{5}$$

where $e \equiv \det e^\alpha_\mu$, the Ricci scalar $\mathcal{R}$ is defined in terms of the Einstein-frame 4-d metric $g_{\mu\nu} = \eta_{\alpha\beta}e^\alpha_\mu e^\beta_\nu$, $G^i_{\mu\nu} \equiv \partial_\mu B^i_\nu - \partial_\nu B^i_\mu$, $F^4_{\mu\nu\,ij} \equiv F'_{\mu\nu\,ij} + G^k_{\mu\nu}A_{ijk}$, and the dots $(\cdots)$ denotes the terms involving the pseudo-scalars A_{ijk} and the two-form fields $A_{\mu\nu\,i}$. Here, $g_{ij} \equiv \eta_{ab}e^a_i e^b_j = -e^a_i e^a_j$ is the internal metric and the curved space indices $(i, j, ...)$ are raised by g^{ij} [5]. The 4-d effective action (5) is manifestly invariant under the $SL(7, \Re)$ target space transformation:

[5]The dilaton field φ and the internal metric g_{ij} in Eq. (5) are related to the dilaton field φ and the unimodular part of the internal metric ρ_{ij}, used in Ref. [24] as $\varphi \to \frac{\sqrt{7}}{3}\varphi$, $g_{ij} \to -e^{\frac{2}{3\sqrt{7}}\varphi}\rho_{ij}$.

$$g_{ij} \to U_{ik} g_{kl} U_{jl}, \quad G^i{}_{\mu\nu} \to (U^{-1})_{ik} G^k{}_{\mu\nu}, \quad F^4_{\mu\nu\, ij} \to (U^{-1})_{ki} (U^{-1})_{lj} F^4_{\mu\nu\, kl}, \tag{6}$$

and the dilaton φ and the 4-d metric $g_{\mu\nu}$ remain intact, where $U \in SL(7, \Re)$.

The $SL(7, \Re)$ target space symmetry can be enlarged to the global $SL(8, \Re)$ symmetry by realizing that 7 scalars φ^i, which are equivalent to $F_{\mu\nu\rho\, i}$ through the duality transformation, $F^{\mu\nu\rho}_{\ \ i} \sim (\sqrt{-g})^{-1} e^{-2\varphi} g_{ij} \varepsilon^{\mu\nu\rho\sigma} \partial_\sigma \varphi^j$, are unified with the Siebenbein e^i_a to form a matrix parameterizing $SL(8, \Re)$ [27]. In this case, 7 KK gauge fields B^k_μ and 21 "magnetic" duals B^{ij}_μ of $A_{\mu\, ij}$ form canonical gauge fields of 28 $U(1)$ groups. With a further inclusion of 35 pseudo-scalar fields A_{ijk}, the $SL(8, \Re)$ group is enlarged to the exceptional group E_7, in which case 28 $U(1)$ gauge fields and their 28 dual fields form the **56** fundamental representation of E_7.

The elements of the above enlarged symmetry groups can be used to provide transformations on existing 4-d solutions, thus generating a family of solutions with the *same* 4-d space-time $g_{\mu\nu}$ and dilaton field φ, however, with transformed dyonic charges and other scalar fields.

In this paper, we find a class of 4-d, supersymmetric, Abelian solutions where all the scalar fields, except the dilaton and the diagonal components of the internal metric, are set to zero. We primarily concentrate on a subset of $SO(7) \subset SL(7, \Re)$ transformation on these basis solutions in order to construct the most general family of solutions with scalar fields other than the internal metric g_{ij} and the dilaton φ turned off. The ultimate goal, however, is to find the generating solutions which, supplemented by a subset of E_7 transformations, would generate *all* the supersymmetric BH solutions with all the scalar fields turned on [6]. We conjecture that *all* the supersymmetric BH's of 11-d SG are generated by imposing a subset of E_7 transformations [7] (of the effective 4-d action) on only *one* type of the generating solution. Such a generating solution would reduce to two separate classes, which are discussed in the next chapter, by taking special limits of charge configurations. On the other hand, in order to generate all the non-extreme BH's, one would have to employ a larger symmetry of the corresponding 3-d effective action for stationary solutions. We speculate that such an enlarged symmetry might be E_8.

---

[6] Within the context of Abelian KK BH's, such a program has been completed. $SO(n)/SO(n-2)$ transformations on the $U(1)_M \times U(1)_E$ supersymmetric solutions [24] generate the most general supersymmetric static BH's [23] in KK theory. On the other hand, in order to generate the most general non-extreme solutions [25], one has to employ a subset of a larger symmetry, *i.e.*, a subset of $SO(2, n) \subset SL(n+2, \Re)$ transformations, which correspond to the symmetry transformations of effective 3-d action of stationary solutions.

[7] In the quantum version with charges quantized in multiples of the elementary electric and magnetic charges, E_7 would be broken down to the corresponding discrete subgroup.

III. FOUR-DIMENSIONAL, SUPERSYMMETRIC, STATIC, SPHERICALLY SYMMETRIC SOLUTIONS OF ELEVEN-DIMENSIONAL SUPERGRAVITY

In this section, we study supersymmetric, static, spherically symmetric configurations arising from two sectors of effective 4-d theory (5):

- The first sector corresponds to configurations with non-zero electric and magnetic charges arising only from gauge fields of Abelian internal isometry group, *i.e.*, KK gauge fields $B^i_\mu \neq 0$. We will refer to this class of solutions as supersymmetric "KK BH's".

- The second sector corresponds to configurations with non-zero electric and magnetic charges only from the Abelian gauge fields $A_{\mu\, ij}$ associated with the 3-form field $A^{(11)}_{MNP}$. We will refer to this class of solutions as supersymmetric "3-form BH's".

Within each class of solutions, we shall obtain the most general solution, with non-zero internal metric g_{ij} and dilaton field φ, while the other scalar fields are turned off.

The spherically symmetric Ansatz for the 4-d space-time metric [8] is chosen to be

$$g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \lambda(r)\mathrm{d}t^2 - \lambda^{-1}(r)\mathrm{d}r^2 - R(r)(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2), \tag{7}$$

and the scalar fields depend on the radial coordinate r only. For a particular $U(1)$ gauge field A_μ, the non-zero components, compatible with the spherical symmetry, are given in the polar coordinate by

$$A_\phi = P(1-\cos\theta), \qquad A_t = \psi(r), \tag{8}$$

where $E(r) = -\partial_r\psi(r) \sim \frac{Q}{r^2}$ $(r\to\infty)$, and P and Q are the physical magnetic and electric charges. Here, the expression for an electric field, *i.e.*, the (t,r) component of the $U(1)$ field strength, in terms of the scalar fields and the 4-d metric components are obtained from the Gauss's law derived from the Lagrangian density (5). The expressions for two types of electric field strengths are given in the following form:

$$\begin{aligned} \nabla_r(e^\varphi g_{ij} G^{j\,rt}) = 0 \quad &\Longrightarrow \quad G^i_{tr} = \frac{g^{ij}\tilde{Q}_j}{Re^\varphi}, \\ \nabla_r(e^\varphi g^{ik} g^{jl} F^{rt}_{kl}) = 0 \quad &\Longrightarrow \quad F_{tr\,ij} = \frac{g_{ik}g_{jl}\tilde{Q}^{kl}}{Re^\varphi} \ \ (\text{with } B^i_\mu = 0), \end{aligned} \tag{9}$$

where the physical electric charges are given by $Q^i = e^{-\varphi_\infty} g^{ij}_\infty \tilde{Q}_j$ and $Q_{ij} = e^{-\varphi_\infty} g_{ik\,\infty} g_{jl\,\infty}\tilde{Q}^{kl}$.

---

[8]The moving frame is then defined in terms of the Vierbein of the following form:

$$e^{\hat t}_t = \lambda^{\frac12}, \quad e^{\hat\theta}_\theta = R^{\frac12}, \quad e^{\hat\phi}_\phi = R^{\frac12}\sin\theta, \quad e^{\hat r}_r = \lambda^{-\frac12},$$

which yields the metric $g_{\mu\nu} = \eta_{\alpha\beta}e^\alpha_\mu e^\beta_\nu$ defined above. Here, $\alpha,\beta = \hat t, \hat\theta, \hat\phi, \hat r$ are flat indices and the flat space-time gamma matrices are ordered in the same manner, *i.e.*, $\gamma^{\hat t} = \gamma^0, ..., \gamma^{\hat r} = \gamma^3$.

A. Kaluza-Klein Black Hole Solutions

The first class of solutions corresponds to supersymmetric KK BH's, *i.e.*, $U(1)$ gauge fields are associated with the isometry group of the internal space. In this case, the gauge fields arising from the 3-form field are turned off, along with all the scalar fields, except the internal metric fields g_{ij} and the dilaton φ. Then, the Lagrangian density (5) reduces to the 4-d effective Lagrangian density of 11-d KK theory compactified on a 7-torus. The corresponding action has a manifest invariance under $SO(7) \subset SL(7, \Re)$ rotations (see Eq.(6)) with the internal metric coefficients g_{ij} transforming as a symmetric **27** representation of $SO(7)$, and the $U(1)$ gauge fields B^i_μ as **7** of $SO(7)$.

Since 4-d, Abelian, supersymmetric BH's of $(4+n)$-d KK theory have been explicitly constructed in Refs. [24,23], here, we summarize the results for the special case of $n = 7$.

With a diagonal internal metric Ansatz, supersymmetric spherically symmetric configurations choose the vacuum where the isometry group of the internal space is broken down to $U(1)_M \times U(1)_E$, *i.e.*, they correspond to a dyonic configuration with magnetic and electric charges coming from different $U(1)$ gauge groups. The most general supersymmetric spherically symmetric configurations with a non-diagonal internal metric g_{ij} are obtained by imposing the $SO(7)/SO(5)$ rotations on the $U(1)_M \times U(1)_E$ solutions. The charge vectors $\vec{\mathcal{P}} \equiv (P_1, ..., P_7)$ and $\vec{\mathcal{Q}} \equiv (Q_1, ..., Q_7)$ of this general class of supersymmetric solutions are constrained by $\vec{\mathcal{P}} \cdot \vec{\mathcal{Q}} = 0$, thereby having $2n - 1 = 13$ degrees of freedom.

Explicit supersymmetric $U(1)_M \times U(1)_E$ solutions of 11-d SG with the *j-th* gauge field magnetic and the *k-th* gauge field electric, and with a diagonal internal metric Ansatz $g_{ij} = \mathrm{diag}(g_{11}, \cdots, g_{77})$, are given by

$$
\begin{aligned}
\lambda &= \frac{r - |\mathbf{P}_{j\,\infty}| - |\mathbf{Q}_{k\,\infty}|}{(r - |\mathbf{P}_{j\,\infty}|)^{\frac{1}{2}}(r - |\mathbf{Q}_{k\,\infty}|)^{\frac{1}{2}}}, \\
R &= r^2\Big(1 - \frac{|\mathbf{P}_{j\,\infty}| + |\mathbf{Q}_{k\,\infty}|}{r}\Big)\Big(1 - \frac{|\mathbf{P}_{j\,\infty}|}{r}\Big)^{\frac{1}{2}}\Big(1 - \frac{|\mathbf{Q}_{k\,\infty}|}{r}\Big)^{\frac{1}{2}}, \\
e^{2(\varphi - \varphi_\infty)} &= \frac{r - |\mathbf{P}_{j\,\infty}|}{r - |\mathbf{Q}_{k\,\infty}|}, \\
\frac{g_{ii}}{g_{ii\,\infty}} &= 1 \qquad (i \neq j, k) \quad , \\
\frac{g_{jj}}{g_{jj\,\infty}} &= \frac{r - |\mathbf{P}_{j\,\infty}| - |\mathbf{Q}_{k\,\infty}|}{(r - |\mathbf{Q}_{k\,\infty}|)}, \\
\frac{g_{kk}}{g_{kk\,\infty}} &= \frac{(r - |\mathbf{P}_{j\,\infty}|)}{r - |\mathbf{P}_{j\,\infty}| - |\mathbf{Q}_{k\,\infty}|}, \\
a^{\mathbf{m}}_u(r) &= a^{\mathbf{m}}_{u\,\infty}\left(\frac{r - |\mathbf{P}_{j\,\infty}| - |\mathbf{Q}_{k\,\infty}|}{r - |\mathbf{P}_{j\,\infty}|}\right)^{\frac{1}{4}},
\end{aligned}
\tag{10}
$$

where $\mathbf{P}_{j\infty} \equiv e^{\frac{1}{2}\varphi_\infty} g^{\frac{1}{2}}_{jj\,\infty} P_j$ and $\mathbf{Q}_{k\infty} \equiv e^{\frac{1}{2}\varphi_\infty} g^{\frac{1}{2}}_{kk\,\infty} Q_k$ are the "screened" magnetic and electric charges (here, P_j and Q_k are the physical magnetic and electric charges of the *j-th* and *k-th* gauge fields). The subscript ∞ refers to the asymptotic ($r \to \infty$) value of the corresponding scalar field. The ADM mass of the configuration is given by $M = |\mathbf{P}_{j\,\infty}| + |\mathbf{Q}_{k\,\infty}|$.

For $P_j \neq 0$ and $Q_k \neq 0$, 4-d space-time has a null singularity, finite temperature ($T_H = (4\pi)^{-1}|\mathbf{P}_{j\infty}\mathbf{Q}_{k\infty}|^{-1/2}$) and zero entropy. If either of P_j or Q_k is set equal to zero, the singularity becomes naked and the temperature diverges.

B. Three-Form Black Hole Solutions

The second class of solutions is static, spherically symmetric configurations associated with the 21 Abelian pseudo-vector fields $A_{\mu\, ij}$, arising from the 3-form fields $A^{(11)}_{MNP}$. In this case, we set the KK gauge fields B^i_μ equal to zero, as well as all the other scalar fields except those associated with the internal metric g_{ij} and the dilaton φ. The corresponding 4-d bosonic effective Lagrangian density is then of the following from:

$$\mathcal{L} = -\frac{1}{4}e[\mathcal{R} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{1}{4}\partial_\mu g_{ij}\partial^\mu g^{ij} + \frac{1}{2}e^{\varphi}g^{ik}g^{jl}F_{\mu\nu\, ij}F^{\mu\nu}{}_{kl}], \tag{11}$$

where $F_{\mu\nu\, ij} \equiv \partial_\mu A_{\nu\, ij} - \partial_\nu A_{\mu\, ij}$. The bosonic action (11) has again a manifest invariance under the $SO(7) \subset SL(7,\Re)$ rotations (see Eq. (6)) with the internal metric coefficients g_{ij} transforming as a symmetric representation **27** of $SO(7)$, and the $U(1)$ gauge fields $A_{\mu\, ij}$ as an antisymmetric representation **21** of $SO(7)$.

1. Killing Spinor Equations

The supersymmetric solutions of the above action are invariant under the gravitino transformations (2), which for the above bosonic field content reduce to the following form:

$$\begin{aligned}
\delta\psi_\mu &= \partial_\mu\varepsilon + \frac{1}{4}\omega_{\mu\beta\gamma}\gamma^{\beta\gamma}\varepsilon - \frac{1}{4}e^\alpha_\mu\eta_{\alpha[\beta}e^\nu_{\gamma]}\partial_\nu\varphi\gamma^{\beta\gamma}\varepsilon + \frac{1}{8}(e^l_b\partial_\mu e_{lc} - e^l_c\partial_\mu e_{lb})\gamma^{bc}\varepsilon \\
&\quad + \frac{i}{24}e^{\frac{\varphi}{2}}F_{\nu\rho\, ij}\gamma^{\nu\rho}{}_\mu\gamma^{ij}\varepsilon - \frac{i}{6}e^{\frac{\varphi}{2}}F_{\mu\nu\, ij}\gamma^\nu\gamma^{ij}\varepsilon, \\
\delta\psi_k &= -\frac{1}{4}e^{\frac{\varphi}{2}}(\partial_\rho e_{kb} + e^c_k e^l_b\partial_\rho e_{lc})\gamma^{\rho 5}\gamma^b\varepsilon + \frac{i}{24}e^\varphi F_{\mu\nu\, ij}\gamma^{\mu\nu 5}\gamma^{ij}{}_k\varepsilon - \frac{i}{6}e^\varphi F_{\mu\nu\, kl}\gamma^{\mu\nu 5}\gamma^l\varepsilon,
\end{aligned} \tag{12}$$

where $\omega_{\mu\beta\gamma}$ is the spin-connection defined in terms of the Vierbein e^α_μ and $[a \cdots b]$ denotes the antisymmetrization of the corresponding indices. For the 11-d gamma matrices, which satisfy the $SO(10,1)$ Clifford algebra $\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}$, we have chosen the following representation:

$$\Gamma^\alpha = \gamma^\alpha \otimes I, \qquad \Gamma^a = \gamma^5 \otimes \gamma^a, \tag{13}$$

where $\{\gamma^\alpha, \gamma^\beta\} = 2\eta^{\alpha\beta}$, $\{\gamma^a, \gamma^b\} = -2\delta^{ab}$, I is the 8×8 identity matrix and $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$. The above representation (13) is compatible with the triangular gauge form (3) ($SO(10,1) \to SO(3,1)\times SO(7)$) of the Elfbein. The gamma matrices with more than one index denote antisymmetric products of the corresponding matrices, *e.g.*, $\gamma^{\alpha\beta} \equiv \gamma^{[\alpha}\gamma^{\beta]}$, and the gamma matrices with curved indices are defined by multiplying with the Vierbein, *e.g.*, $\gamma^\mu \equiv e^\mu_\alpha\gamma^\alpha$. Correspondingly, the spinor index A of an 11-d spinor ε^A is decomposed into $A = (\mathbf{a}, \mathbf{m})$, *i.e.*, $\varepsilon^A = \varepsilon^{(\mathbf{a},\mathbf{m})}$, where $\mathbf{a} = 1, ..., 4$ is the spinor index for a four component 4-d spinor and $\mathbf{m} = 1, ..., 8$ is the index for the spinor representation of the group $SO(7)$.

With a choice of spherical Ansätze given by (7) through (9), the Killing spinor equations, which are obtained by setting the gravitino supersymmetry transformation (12) equal to zero, are of the following form:

$$\frac{1}{4}\lambda'\gamma^{03}\varepsilon - \frac{1}{4}\lambda\partial_r\varphi\gamma^{03}\varepsilon + \frac{i}{12}\frac{e^i_a e^j_b P_{ij}}{Re^{-\frac{\varphi}{2}}}\lambda^{\frac{1}{2}}\gamma^{120}\gamma^{ab}\varepsilon - \frac{i}{6}\frac{e^a_i e^b_j Q^{ij}}{Re^{\frac{\varphi}{2}}}\lambda^{\frac{1}{2}}\gamma^3\gamma^{ab}\varepsilon = 0, \tag{14}$$

$$\partial_\theta\varepsilon - \frac{1}{4}\lambda^{\frac{1}{2}}\frac{R'}{R^{1/2}}\gamma^{13}\varepsilon + \frac{1}{4}\sqrt{\lambda R}\partial_r\varphi\gamma^{13}\varphi - \frac{i}{12}\frac{e^a_i e^b_j Q^{ij}}{Re^{\frac{\varphi}{2}}}R^{\frac{1}{2}}\gamma^{031}\gamma^{ab}\varepsilon - \frac{i}{6}\frac{e^i_a e^j_b P_{ij}}{Re^{-\frac{\varphi}{2}}}R^{\frac{1}{2}}\gamma^2\gamma^{ab}\varepsilon = 0, \tag{15}$$

$$\begin{aligned}\partial_\phi\varepsilon - \frac{1}{2}\cos\theta\gamma^{21}\varepsilon - \frac{1}{4}\lambda^{\frac{1}{2}}\frac{R'}{R^{1/2}}\sin\theta\gamma^{23}\varepsilon + \frac{1}{4}\sqrt{\lambda R}\sin\theta\partial_r\varphi\gamma^{23}\varepsilon - \frac{i}{12}\frac{e^a_i e^b_j Q^{ij}}{Re^{\frac{\varphi}{2}}}R^{\frac{1}{2}}\sin\theta\gamma^{032}\gamma^{ab}\varepsilon \\ + \frac{i}{6}\frac{e^i_a e^j_b P_{ij}}{Re^{-\frac{\varphi}{s}}}R^{\frac{1}{2}}\sin\theta\gamma^1\gamma^{ab}\varepsilon = 0,\end{aligned} \tag{16}$$

$$\partial_r\varepsilon + \frac{1}{8}(e^l_b\partial_r e_{lc} - e^l_c\partial_r e_{lb})\gamma^{bc}\varepsilon - \frac{i}{12}\frac{e^i_a e^j_b P_{ij}}{Re^{-\frac{\varphi}{2}}}\lambda^{-\frac{1}{2}}\gamma^{123}\gamma^{ab}\varepsilon + \frac{i}{6}\frac{e^a_i e^b_j Q^{ij}}{Re^{\frac{\varphi}{2}}}\lambda^{-\frac{1}{2}}\gamma^0\gamma^{ab}\varepsilon = 0, \tag{17}$$

$$\begin{aligned}-\frac{1}{4}(e^k_d\,\partial_r\,e_{kb} + e^k_b\partial_r e_{kd})\lambda^{\frac{1}{2}}\gamma^{35}\gamma^b\varepsilon + \frac{i}{12}\frac{e^a_i e^b_j Q^{ij}}{Re^{\frac{\varphi}{2}}}\gamma^{035}\gamma^{ab}{}_d\varepsilon + \frac{i}{12}\frac{e^i_a e^j_b P_{ij}}{Re^{-\frac{\varphi}{2}}}\gamma^{125}\gamma^{ab}{}_d\varepsilon \\ - \frac{i}{3}\frac{e^d_k e^b_l Q^{kl}}{Re^{\frac{\varphi}{2}}}\gamma^{035}\gamma^b\varepsilon - \frac{i}{3}\frac{e^k_d e^l_b P_{kl}}{Re^{-\frac{\varphi}{2}}}\gamma^{125}\gamma^b\varepsilon = 0,\end{aligned} \tag{18}$$

where the last equation (18) is obtained from $e^k_d\delta\psi_k = 0$ and the prime denotes the differentiation with respect to r.

2. Constraints on Charges

Before attempting to solve the Killing spinor equations, we would like to derive the constraints on charges P_{ij} and Q^{ij} for a general supersymmetric configuration.

First, one has to determine, by now, a standard form of the angular coordinate (θ, ϕ) dependence of the spinors $\varepsilon^{\mathbf{m}}$ for static, spherically symmetric configurations. One multiplies (15) by $\gamma^1\sin\theta$ and (16) by γ^2, subtracts the two resultant equations, and multiplies the result by γ^2, thus yielding the following equation:

$$[2\partial_\phi + \gamma^1\gamma^2\cos\theta - 2(\gamma^1\gamma^2\sin\theta)\partial_\theta]\varepsilon^{\mathbf{m}} = 0, \tag{19}$$

which fixes the angular coordinate dependence of the spinors to be

$$(\varepsilon^{1,\mathbf{m}}_{u,\ell}, \varepsilon^{2,\mathbf{m}}_{u,\ell}) = e^{i\sigma^2\theta/2}e^{i\sigma^3\phi/2}(a^{1,\mathbf{m}}_{u,\ell}(r), a^{2,\mathbf{m}}_{u,\ell}(r)), \tag{20}$$

where $\varepsilon^{\mathbf{m}}_{u,\ell}$ are the upper (or lower) two components of the four component spinor $\varepsilon^{\mathbf{m}}$, *i.e.*, $(\varepsilon^{\mathbf{m}})^T = (\varepsilon^{\mathbf{m}}_u, \varepsilon^{\mathbf{n}}_\ell)$, and $a^{\mathbf{m}}_{u,\ell}(r)$ are the corresponding two component spinors that depend on the radial coordinate r only.

Then, the Killing spinor equations (14), (15) and (18), supplemented by (20), assume the following form:

$$\frac{1}{\sqrt{\lambda}}[4\sqrt{\lambda R} - 2\lambda\partial_r R - R\partial_r\lambda + 3\lambda R\partial_r\varphi]\varepsilon_{u,\ell} = \pm\mathbf{P}_{ab}\gamma^{ab}\varepsilon_{\ell,u}, \tag{21}$$

$$\frac{1}{\sqrt{\lambda}}[2\sqrt{\lambda R} - \lambda\partial_r R - 2R\partial_r\lambda + 3\lambda R\partial_r\varphi]\varepsilon_{u,\ell} = i\mathbf{Q}_{ab}\gamma^{ab}\varepsilon_{\ell,u}, \tag{22}$$

$$\frac{1}{4}\mathcal{A}_{db}\gamma^b\varepsilon_{\ell,u} \pm \frac{1}{12}(\mathbf{P}_{ab} \mp i\mathbf{Q}_{ab})\gamma^{ab}{}_d\varepsilon_{u,\ell} \mp \frac{1}{3}(\mathbf{P}_{db} \mp i\mathbf{Q}_{db})\gamma^b\varepsilon_{u,\ell} = 0, \tag{23}$$

where $\mathbf{P}_{ab} \equiv e^{\frac{\varphi}{2}}e^i_a e^j_b P_{ij}$, $\mathbf{Q}_{ab} \equiv e^{-\frac{\varphi}{2}}e^a_i e^b_j Q^{ij}$, and $\mathcal{A}_{ab} \equiv R\lambda^{\frac{1}{2}}(e^k_a\partial_r e_{kb} + e^k_b\partial_r e_{ka})$. Here, the upper [lower] signs of the equations are associated with the first [second] subscripts of the two component spinors.

We are now able to derive constraints on charges P_{ij} and Q^{ij}. These constraints are obtained by ensuring that the gravitino Killing spinor equations (21) and (22), which do not explicitly depend on the radial derivatives of the Siebenbein, are satisfied. After a suitable manipulation [9] of (21) and (22), and using the corresponding anti-commutation relations of γ^{ab} matrices, *i.e.*, $\{\gamma^{ab}, \gamma^{cd}\} = 2\gamma^{abcd} - 2(\eta^{ad}\eta^{bc} - \eta^{ac}\eta^{bd})$, one arrives at the following set of the first order differential equations

$$\begin{aligned}
&\frac{1}{\sqrt{\lambda}}[4\sqrt{\lambda R} - 2\lambda\partial_r R - R\partial_r\lambda + 3\lambda R\partial_r\varphi] = 2\eta_P[\sum_{a<b}(\mathbf{P}_{ab})^2]^{1/2},\\
&\frac{1}{\sqrt{\lambda}}[2\sqrt{\lambda R} - \lambda\partial_r R - 2R\partial_r\lambda + 3\lambda R\partial_r\varphi] = 2\eta_Q[\sum_{a<b}(\mathbf{Q}_{ab})^2]^{1/2} \quad (\eta_{P,Q} \equiv \pm 1),
\end{aligned} \tag{24}$$

and the following constraints on the charge configuration:

$$\sum \mathbf{P}_{ab}\mathbf{P}_{cd}\gamma^{abcd} = 0 = \sum \mathbf{Q}_{ab}\mathbf{Q}_{cd}\gamma^{abcd}, \qquad \sum_{i\neq j} P_{ij}Q^{ij} = 0 = \sum \mathbf{P}_{ab}\mathbf{Q}_{cd}\gamma^{abcd} \quad , \tag{25}$$

as well as the constraint between the upper and lower two-component spinors:

$$\varepsilon_u = \eta_P[4\sum_{a'<b'}(\mathbf{P}_{a'b'})^2]^{-1/2}\mathbf{P}_{ab}\gamma^{ab}\varepsilon_\ell \quad (\eta_P \equiv \pm 1). \tag{26}$$

We can find a constraint on charges by analyzing the constraints (25) on the asymptotic values $\mathbf{P}_{ab\,\infty}$ and $\mathbf{Q}_{ab\,\infty}$ of $\mathbf{P}_{ab}$ and $\mathbf{Q}_{ab}$. Manifest $SL(7,\Re)$ symmetry, *i.e.*, the rescaling symmetry and the $SO(7)$ rotations, allows one to bring the asymptotic value of the internal metric to the form $g_{ij\,\infty} = -\delta_{ij}$, without loss of generality. Thus, the asymptotic value of the Siebenbein can be chosen to be $e^i_{a\,\infty} \equiv \delta^i_a$, up to global $SO(7)$ rotations and the rescaling of the radii of the internal coordinates. In the case of $e^i_{a\,\infty} \equiv \delta^i_a$, the only charge configurations, which satisfy the constraints (25), are of the following two types:

---

[9]The manipulation is similar to the one of Ref. [23].

- The only non-zero charges are $P_{ij} \neq 0$ and $Q^{ij} \neq 0$, subject to the constraint $\sum_j P_{ij}Q^{ij} = 0$, where i corresponds to a fixed choice of the index.
- The only allowed nonzero charges are $(P_{ij}, P_{ik}, P_{jk}) \neq 0$ and $(Q^{ij}, Q^{ik}, Q^{jk}) \neq 0$, subject to the constraint $P_{ij}Q^{ij} + P_{ik}Q^{ik} + P_{jk}Q^{jk} = 0$, where $i \neq j \neq k$ correspond to a fixed choice of three indices.

The first charge configuration can be transformed, by a subset of $SO(6) \subset SO(7)$ rotations, into a form in which the only nonzero charges are $P_{ij} \neq 0$ and $Q^{ik} \neq 0$, where $i \neq j \neq k$ correspond to a fixed choice of the three indices. The second charge configuration can also be transformed, by a subset of $SO(3) \subset SO(7)$ rotations, in the same form with only nonzero charges given by $P_{ij} \neq 0$ and $Q^{ik} \neq 0$.

A generating solution is, therefore, the one with one electric charge, say Q^{ij}, and one magnetic charge, say P_{ik}. Thus, the most general supersymmetric configuration can be obtained by imposing a subset of global $SO(7)$ transformations, *i.e.*, $SO(7)/SO(3)$ transformations with $3n-6 = 15$ parameters, on a generating solution with one electric charge, say Q^{ij}, and one magnetic charge, say P_{ik} (2 parameters). Consequently, the most general supersymmetric charge configuration is of *constrained* one; among non-zero $n(n-1)/2 = 21$ electric charges and $n(n-1)/2 = 21$ magnetic charges, only $(3n-6)+2 = 17$ are independent.

3. Supersymmetric Three-Form Black Hole Solutions with Diagonal Internal Metric

Our next goal is to obtain the explicit form of the generating solution with only one magnetic charge P_{ij} and one electric charge Q^{ik} non-zero. This type of solution will be obtained with a diagonal internal metric Ansatz, *i.e.*, the Siebenbein is chosen to be of the form:

$$e_a^k = \mathrm{diag}(e_1, ..., e_7). \tag{27}$$

In this case, the first order differential equations (21) $-$ (23) reduce to the following simplified form:

$$\frac{1}{\sqrt{\lambda}}[4\sqrt{\lambda R} - 2\lambda\partial_r R - R\partial_r\lambda + 3\lambda R\partial_r\varphi] = 2\eta_P \mathbf{P}_{\hat{i}\hat{j}}, \tag{28}$$

$$\frac{1}{\sqrt{\lambda}}[2\sqrt{\lambda R} - \lambda\partial_r R - 2R\partial_r\lambda + 3\lambda R\partial_r\varphi] = 2\eta_Q \mathbf{Q}_{\hat{i}\hat{k}}, \tag{29}$$

$$-\frac{1}{4}\mathcal{A}_{\hat{i}\hat{i}}\varepsilon_{\ell,u} \mp \frac{1}{3}\mathbf{P}_{\hat{i}\hat{j}}\Gamma^{\hat{i}\hat{j}}\varepsilon_{u,\ell} + \frac{i}{3}\mathbf{Q}_{\hat{i}\hat{k}}\Gamma^{\hat{i}\hat{k}}\varepsilon_{u,\ell} = 0, \tag{30}$$

$$-\frac{1}{4}\mathcal{A}_{\hat{j}\hat{j}}\varepsilon_{\ell,u} \mp \frac{1}{3}\mathbf{P}_{\hat{i}\hat{j}}\Gamma^{\hat{i}\hat{j}}\varepsilon_{u,\ell} - \frac{i}{6}\mathbf{Q}_{\hat{i}\hat{k}}\Gamma^{\hat{i}\hat{k}}\varepsilon_{u,\ell} = 0, \tag{31}$$

$$-\frac{1}{4}\mathcal{A}_{\hat{k}\hat{k}}\varepsilon_{\ell,u} \pm \frac{1}{6}\mathbf{P}_{\hat{i}\hat{j}}\Gamma^{\hat{i}\hat{j}}\varepsilon_{u,\ell} + \frac{i}{3}\mathbf{Q}_{\hat{i}\hat{k}}\Gamma^{\hat{i}\hat{k}}\varepsilon_{u,\ell} = 0, \tag{32}$$

$$-\frac{1}{4}\mathcal{A}_{\hat{\ell}\hat{\ell}}\varepsilon_{\ell,u} \pm \frac{1}{6}\mathbf{P}_{\hat{i}\hat{j}}\Gamma^{\hat{i}\hat{j}}\varepsilon_{u,\ell} - \frac{i}{6}\mathbf{Q}_{\hat{i}\hat{k}}\Gamma^{\hat{i}\hat{k}}\varepsilon_{u,\ell} = 0 \qquad (\hat{\ell} \neq \hat{i}, \hat{j}, \hat{k}), \tag{33}$$

where $\mathcal{A}_{\hat{n}\hat{n}} \equiv 2R\lambda^{\frac{1}{2}}\partial_r \mathrm{ln} e_{\hat{n}}$ and the hats on the curved indices denote the corresponding flat indices.

We shall now solve the equations (28) $-$ (33) to get the explicit dyonic supersymmetric solutions with one magnetic P_{ij} and one electric Q_{ik} charges [10]. First, from (30) $-$ (33), we obtain the following relations among Siebenbein components:

$$\check{e}_{\hat{i}} = (\check{e}_{\hat{\ell}})^{-2}, \qquad \check{e}_{\hat{j}}\check{e}_{\hat{k}}\check{e}_{\hat{\ell}} = 1, \tag{34}$$

where $\check{e}_{\hat{i}} \equiv e_{\hat{i}}/e_{\hat{i}\,\infty}$. Substituting the difference between (30) and (31) into (29), we obtain

$$\check{e}_{\hat{i}}^{2}\check{e}_{\hat{j}}^{-2} = e^{3(\varphi-\varphi_\infty)}\lambda^{-1}. \tag{35}$$

Making use of (34), we arrive at the relation

$$e^{(\varphi-\varphi_\infty)} = \det \check{e}^{d}_{m} = \check{e}_{\hat{\ell}}^{-1} \quad (\hat{\ell} \neq \hat{i}, \hat{j}, \hat{k}). \tag{36}$$

Then, the following relation between the 4-d metric components $\lambda(r)$ and $R(r)$ can be obtained by substituting (28) and (29) into (33), making use of (36):

$$2\sqrt{\lambda R} = \lambda\partial_r R + R\partial_r \lambda, \tag{37}$$

which can be solved to yield

$$\lambda R = (r - r_H)^2, \tag{38}$$

where the integration constant r_H corresponds to the event horizon, *i.e.*, $\lambda(r_H) = 0$. This is, again, another standard result for 4-d, supersymmetric, spherically symmetric, static solutions.

Furthermore, with the help of (37) we can simplify (28) and (29) to the following forms:

$$\sqrt{\lambda}R(\frac{\partial_r\lambda}{\lambda} + 3\partial_r\varphi) = 2\eta_P\mathbf{P}_{\hat{i}\hat{j}}, \tag{39}$$

$$\sqrt{\lambda}R(-\frac{\partial_r\lambda}{\lambda} + 3\partial_r\varphi) = 2\eta_Q\mathbf{Q}_{\hat{i}\hat{k}}. \tag{40}$$

Note the symmetry of the above two equations under the electric-magnetic duality transformation, *i.e.*, $\mathbf{P}_{\hat{i}\hat{j}} \leftrightarrow -\mathbf{Q}_{\hat{i}\hat{k}}$ and $\varphi \to -\varphi$. By adding these two equations, we obtain the following equation

---

[10] The derivation for supersymmetric as well as non-extreme solutions are along the similar lines as those for $U(1)_M \times U(1)_E$ supersymmetric [24] and non-extreme [28,29,25] KK BH's.

$$\partial_r \varphi = \frac{1}{3\sqrt{\lambda}R}[\eta_P \mathbf{P}_{\hat{i}\hat{j}} + \eta_Q \mathbf{Q}_{\hat{i}\hat{k}}], \tag{41}$$

which is in accordance with a no-hair theorem, *i.e.*, when electromagnetic fields are zero ($P_{ij} = 0 = Q_{ik}$) the dilaton field φ is constant.

The expression relating the 4-d metric component λ and the dilaton φ can be obtained by multiplying (39) by $\eta_Q \mathbf{Q}_{\hat{i}\hat{k}}$ and (40) by $\eta_P \mathbf{P}_{\hat{i}\hat{j}}$, followed by addition of the resulting two equations. The resultant equation can be solved to yield

$$\lambda = \frac{\eta_P e^{3\varphi} \mathbf{P}_{ij\,\infty} + \eta_Q e^{-3\varphi} \mathbf{Q}_{ik\,\infty}}{\eta_P \mathbf{P}_{ij\,\infty} + \eta_Q \mathbf{Q}_{ik\,\infty}}, \tag{42}$$

where $\mathbf{P}_{ij\,\infty} \equiv e^{\varphi_\infty/2} g_{ii\,\infty}^{-\frac{1}{2}} g_{jj\,\infty}^{-\frac{1}{2}} P_{ij}$ and $\mathbf{Q}_{ik\,\infty} \equiv e^{\varphi_\infty/2} g_{ii\,\infty}^{-\frac{1}{2}} g_{kk\,\infty}^{-\frac{1}{2}} Q_{ik}$ are the "screened" electric and magnetic charges. Finally, the following ordinary differential equation for φ is obtained by substituting (42) into (41) and making use of Eqs. (34) $-$ (38):

$$\partial_r \varphi = \frac{1}{3(r - r_H)}[\eta_P e^{3\varphi} \mathbf{P}_{ij\,\infty} + \eta_Q e^{-3\varphi} \mathbf{Q}_{ik\,\infty}]^2 \frac{1}{\eta_P \mathbf{P}_{ij\,\infty} + \eta_Q \mathbf{Q}_{ik\,\infty}}. \tag{43}$$

Note, again, the symmetry of (43) under the electric-magnetic duality transformation. The explicit solution for the dilaton φ is, then, given by

$$e^{3\varphi} = \left(\frac{r - r_H - 2\eta_Q \mathbf{Q}_{ik\,\infty}}{r - r_H + 2\eta_P \mathbf{P}_{ij\,\infty}}\right)^{\frac{1}{2}} = \left(\frac{r - 2|\mathbf{P}_{ij\,\infty}|}{r - 2|\mathbf{Q}_{ik\,\infty}|}\right)^{\frac{1}{2}}, \tag{44}$$

where we have identified $r_H = 2\eta_P \mathbf{P}_{ij\,\infty} - 2\eta_Q \mathbf{Q}_{ik\,\infty}$, and chosen the signs of $\eta_{P,Q}$ so that $\eta_P \mathbf{P}_{ij\,\infty} = |\mathbf{P}_{ij\,\infty}|$ and $-\eta_Q \mathbf{Q}_{ik\,\infty} = |\mathbf{Q}_{ik\,\infty}|$.

Now, we are ready to write down the explicit supersymmetric BH solutions. Making use of the solution (44) for the dilaton φ and the relations (34) $-$ (36) and (42) among various fields, as well as Eq. (18) we obtain the following result for the scalar fields as well as for the Killing spinor [11]:

$$\begin{aligned}
\lambda &= \frac{r - 2|\mathbf{P}_{ij\,\infty}| - 2|\mathbf{Q}_{ik\,\infty}|}{(r - 2|\mathbf{P}_{ij\,\infty}|)^{\frac{1}{2}} (r - 2|\mathbf{Q}_{ik\,\infty}|)^{\frac{1}{2}}}, \\
R &= r^2 (1 - \frac{2|\mathbf{P}_{ik\,\infty}| + 2|\mathbf{Q}_{ik\,\infty}|}{r})(1 - \frac{2|\mathbf{P}_{ij\,\infty}|}{r})^{\frac{1}{2}} (1 - \frac{2|\mathbf{Q}_{ik\,\infty}|}{r})^{\frac{1}{2}}, \\
e^{3(\varphi - \varphi_\infty)} &= \left(\frac{r - 2|\mathbf{P}_{ij\,\infty}|}{r - 2|\mathbf{Q}_{ik\,\infty}|}\right)^{\frac{1}{2}}, \\
g_{ii}/g_{ii\,\infty} = e^{-4(\varphi - \varphi_\infty)} &= \left(\frac{r - 2|\mathbf{Q}_{ik\,\infty}|}{r - 2|\mathbf{P}_{ij\,\infty}|}\right)^{\frac{2}{3}},
\end{aligned}$$

[11] Note, the 3-form BH solution has charges which are different from those of KK BH's by a factor of 2 because of a different normalization for the corresponding gauge fields.

$$
\begin{aligned}
g_{jj}/g_{jj\,\infty} &= \lambda^{-1}e^{-(\varphi-\varphi_\infty)} = \frac{(r-2|\mathbf{P}_{ij\,\infty}|)^{\frac{1}{3}}(r-2|\mathbf{Q}_{ik\,\infty}|)^{\frac{2}{3}}}{r-2|\mathbf{P}_{ij\,\infty}|-2|\mathbf{Q}_{ik\,\infty}|}, \\
g_{kk}/g_{kk\,\infty} &= \lambda e^{-(\varphi-\varphi_\infty)} = \frac{r-2|\mathbf{P}_{ij\,\infty}|-2|\mathbf{Q}_{ik\,\infty}|}{(r-2|\mathbf{P}_{ij\,\infty}|)^{\frac{2}{3}}(r-2|\mathbf{Q}_{ik\,\infty}|)^{\frac{1}{3}}}, \\
g_{\ell\ell}/g_{\ell\ell\,\infty} &= e^{2(\varphi-\varphi_\infty)} = \left(\frac{r-2|\mathbf{P}_{ij\,\infty}|}{r-2|\mathbf{Q}_{ik\,\infty}|}\right)^{\frac{1}{3}} \quad (\ell \neq i,j,k), \\
a^{\mathbf{m}}_u(r) &= a^{\mathbf{m}}_{u\,\infty}\frac{(r-2|\mathbf{P}_{ij\,\infty}|-2|\mathbf{Q}_{ik\,\infty}|)^{\frac{1}{4}}}{(r-2|\mathbf{P}_{ij\,\infty}|)^{\frac{1}{6}}(r-2|\mathbf{Q}_{ik\,\infty}|)^{\frac{1}{12}}},
\end{aligned} \tag{45}
$$

where, again, $\mathbf{P}_{ij\,\infty} \equiv e^{\varphi_\infty/2}g_{ii\,\infty}^{-\frac{1}{2}}g_{jj\,\infty}^{-\frac{1}{2}}P_{ij}$ and $\mathbf{Q}_{ik\,\infty} \equiv e^{\varphi_\infty/2}g_{ii\,\infty}^{-\frac{1}{2}}g_{kk\,\infty}^{-\frac{1}{2}}Q_{ik}$. The 4-d space-time of these solutions is the same as the one of supersymmetric $U(1)_M \times U(1)_E$ KK BH's (see Eq. (10)), provided one makes the replacement $2\mathbf{P}_{\mathbf{ij}\,\infty} \to \mathbf{P}_{j\,\infty}$ and $2\mathbf{Q}_{ik\,\infty} \to \mathbf{Q}_{k\,\infty}$. Thus, the corresponding global space-time and thermal properties are the same.

These supersymmetric solutions saturate the corresponding Bogomol'nyi bound on the ADM mass. The derivation of the bound is along the lines spelled out in Refs. [17,24].

4. Non-extreme 3-Form Black Hole Solutions with Diagonal Internal Metric

We also point out that the corresponding non-extreme solutions can be obtained by solving explicitly the corresponding second order differential equations with a diagonal internal metric Ansatz. Here, we quote the result:

$$
\begin{aligned}
\lambda &= \frac{(r-r_+)}{(r-r_++2\hat{\mathbf{P}}_{ij\,\infty})^{\frac{1}{2}}(r-r_++2\hat{\mathbf{Q}}_{ik\,\infty})^{\frac{1}{2}}}, \\
R &= r^2\Big(1-\frac{r_+-2\beta}{r}\Big)\Big(1-\frac{r_+-2\hat{\mathbf{P}}_{ij\,\infty}}{r}\Big)^{\frac{1}{2}}\Big(1-\frac{r_+-2\hat{\mathbf{Q}}_{ik\,\infty}}{r}\Big)^{\frac{1}{2}}, \\
e^{3(\varphi-\varphi_\infty)} &= \left(\frac{r-r_++2\hat{\mathbf{Q}}_{ik\,\infty}}{r-r_++2\hat{\mathbf{P}}_{ij\,\infty}}\right)^{\frac{1}{2}}, \\
g_{ii}/g_{ii\,\infty} &= \left(\frac{r-r_++2\hat{\mathbf{P}}_{ij\,\infty}}{r-r_++2\hat{\mathbf{Q}}_{ik\,\infty}}\right)^{\frac{2}{3}}, \\
g_{jj}/g_{jj\,\infty} &= \frac{(r-r_++2\hat{\mathbf{P}}_{ij\,\infty})^{\frac{1}{3}}(r-r_++2\hat{\mathbf{Q}}_{ik\,\infty})^{\frac{2}{3}}}{(r-r_+)}, \\
g_{kk}/g_{kk\,\infty} &= \frac{(r-r_+)}{(r-r_++2\hat{\mathbf{P}}_{ij\,\infty})^{\frac{2}{3}}(r-r_++2\hat{\mathbf{Q}}_{ik\,\infty})^{\frac{1}{3}}}, \\
g_{\ell\ell}/g_{\ell\ell\,\infty} &= \left(\frac{r-r_++2\hat{\mathbf{Q}}_{ik\,\infty}}{r-r_++2\hat{\mathbf{P}}_{ij\,\infty}}\right)^{\frac{1}{3}} (\ell \neq i,j,k),
\end{aligned} \tag{46}
$$

where $(\mathbf{P}_{ij\,\infty})^2 = \hat{\mathbf{P}}_{ij\,\infty}(\hat{\mathbf{P}}_{ij\,\infty}-\beta)$, $(\mathbf{Q}_{ik\,\infty})^2 = \hat{\mathbf{Q}}_{ik\,\infty}(\hat{\mathbf{Q}}_{ik\,\infty}-\beta)$, $r_+ = \beta + (|\mathbf{P}_{ij\,\infty}|\sqrt{\beta^2+4\mathbf{P}^2_{ij\,\infty}} - |\mathbf{Q}_{ik\,\infty}|\sqrt{\beta^2+4\mathbf{Q}^2_{ik\,\infty}})/(|\mathbf{P}_{ij\,\infty}|-|\mathbf{Q}_{ik\,\infty}|)$, and $\beta > 0$ is the non-

extremality parameter. The ADM mass M of the configuration, the Hawking temperature T_H, and the entropy S are given by:

$$M = 2\beta + \sqrt{4\mathbf{P}^2_{ij\,\infty} + \beta^2} + \sqrt{4\mathbf{Q}^2_{ik\,\infty} + \beta^2}, \tag{47}$$

$$T_H = 1/(4\pi[\beta + (4\mathbf{P}^2_{ij\,\infty} + \beta^2)^{\frac{1}{2}}]^{\frac{1}{2}}[\beta + (4\mathbf{Q}^2_{ik\,\infty} + \beta^2)^{\frac{1}{2}}]^{\frac{1}{2}}), \tag{48}$$

$$S = 2\pi\beta[\beta + (4\mathbf{P}^2_{ij\,\infty} + \beta^2)^{\frac{1}{2}}]^{\frac{1}{2}}[\beta + (4\mathbf{Q}^2_{ik\,\infty} + \beta^2)^{\frac{1}{2}}]^{\frac{1}{2}}. \tag{49}$$

The global space-time is that of Schwarzschild BH's, with a horizon at $r = r_+$ and a space-like singularity at $r = r_+ - 2\beta$. The 4-d space-time of the above solution is the same as the one of non-extreme $U(1)_M \times U(1)_E$ KK BH's [28], with the replacement $2\mathbf{P}_{ij\,\infty} \to \mathbf{P}_{j\,\infty}$ and $2\mathbf{Q}_{ik\,\infty} \to \mathbf{Q}_{k\,\infty}$.

5. Symmetry Tranformations between Kaluza-Klein and Three-Form Black Hole Solutions

Solutions for supersymmetric (see Eq. (45)) and non-extreme (see Eq. (46)) 3-form BH's with a diagonal internal metric bear similarities with the corresponding solutions for supersymmetric (see Eq. (10)) and non-extreme [28] KK BH's. In particular, the 4-d metric $g_{\mu\nu}$ and thus the global space-time as well as thermal properties of both types of BH's are the same. This is an indication that the two types of the solutions are related by a discrete subset of E_7 transformation. A discrete symmetry, which transforms the effective action of $U(1)_M \times U(1)_E$ KK BH's into the corresponding one for the 3-form BH's, relates the charges P_{ij}, Q^{ik} and the normalized (diagonal) metric coefficients $\breve{g}'_{ii} \equiv g'_{ii}/g'_{ii\,\infty}$ of the 3-form effective action to charges P_j, Q^k and the normalized (diagonal) metric coefficients $\breve{g}_{ii} \equiv g_{ii}/g_{ii\,\infty}$ of the KK effective action in the following way:

$$P_{ij} = \frac{P_j}{2}, \qquad Q^{ik} = \frac{Q^k}{2},$$
$$\frac{\breve{g}'_{jj}}{\breve{g}'_{kk}} = \frac{\breve{g}_{kk}}{\breve{g}_{jj}}, \quad \breve{g}'_{jj}\breve{g}'_{kk} = (\breve{g}_{kk}\breve{g}_{jj})^{-1/3}\breve{g}'_{ii} = (\breve{g}_{jj}\breve{g}_{kk})^{-2/3}, \quad \prod_{\ell\neq(ijk)} \breve{g}'_{\ell\ell} = (\breve{g}_{jj}\breve{g}_{kk})^{4/3}. \tag{50}$$

The above discrete transformation in turn corresponds to a subset of continuous transformations, which preserve the "length" of a combined KK and 3-form "charge vector", and which involve the psueudoscalar field as well [12].

---

[12] A subset of such transformations relates the fields within NS-NS sector of type-IIA string compactified on a 6-torus (see the subsequent chapter), which is the same as a subsector of heterotic string (associated with 6 $U(1)$ KK gauge fields and 6 $U(1)$ gauge fields originating from the 10-d two-form field). In this subsector, the fields are related by the continuous $SO(6,6)$ transformations [20] which also involve 6 pseudo-scalar fields.

IV. TYPE-IIA SUPERSTRING THEORY AND ELEVEN-DIMENSIONAL SUPERGRAVITY

In this chapter, we relate the 4-d supersymmetric BH solutions of 11-d SG, which were discussed in the previous chapter, to the corresponding BH's of type-IIA superstring theory compactified on a 6 torus. In particular, we would like to relate the scalar field degrees of 11-d SG, parameterizing the internal space-time, to the string coupling of the type-IIA superstring compactified on a 6-torus, and thus, by virtue of duality between the two theories, obtain information on mass spectrum of strongly coupled type-IIA string compactified on a 6-torus.

The zero slope limit of the type-IIA 10-d superstring theory can be obtained by dimensional reduction of 11-d SG on a circle S^1 [30]. This can be accomplished by choosing the following triangular gauge form for the Elfbein $E^{(11)\,A}_M$:

$$E^{(11)\,A}_M = \begin{pmatrix} e^{-\frac{\Phi}{3}} e^{(10)\,\breve{\alpha}}_{\breve{\mu}} & e^{\frac{2}{3}\Phi} B_{\breve{\mu}} \\ 0 & e^{\frac{2}{3}\Phi} \end{pmatrix}, \tag{51}$$

where Φ corresponds to the 10-d dilaton field in NS-NS sector of the superstring theory, $e^{(10)\,\breve{\alpha}}_{\breve{\mu}}$ is the Zehnbein in NS-NS sector, and $B_{\breve{\mu}}$ corresponds to a one-form in RR sector. Here, the breve denotes the 10-d space-time vector index. And the 3-form $A^{(11)}_{MNP}$ is decomposed into $A^{(11)}_{MNP} = (A_{\breve{\mu}\breve{\nu}\breve{\rho}}, A_{\breve{\mu}\breve{\nu}11} \equiv A_{\breve{\mu}\breve{\nu}})$, where $A_{\breve{\mu}\breve{\nu}\breve{\rho}}$ is identified as a 3-form in RR sector and $A_{\breve{\mu}\breve{\nu}}$ is the antisymmetric tensor in NS-NS sector. Then, 11-d bosonic action (1) becomes the following 10-d, $N = 2$ SG action:

$$\mathcal{L} = \mathcal{L}_{NS} + \mathcal{L}_R, \tag{52}$$

with

$$\begin{aligned} \mathcal{L}_{NS} &= -\frac{1}{4} e^{(10)} e^{-2\Phi} [\mathcal{R} + 4\partial_{\breve{\mu}}\Phi\partial^{\breve{\mu}}\Phi - \frac{1}{3} F_{\breve{\mu}\breve{\nu}\breve{\rho}} F^{\breve{\mu}\breve{\nu}\breve{\rho}}], \\ \mathcal{L}_R &= -\frac{1}{4} e^{(10)} [\frac{1}{4} G_{\breve{\mu}\breve{\nu}} G^{\breve{\mu}\breve{\nu}} + \frac{1}{12} F'_{\breve{\mu}\breve{\nu}\breve{\rho}\breve{\sigma}} F'^{\,\breve{\mu}\breve{\nu}\breve{\rho}\breve{\sigma}} - \frac{6}{(12)^3} \varepsilon^{\breve{\mu}_1 \cdots \breve{\mu}_{10}} F_{\breve{\mu}_1 \cdots \breve{\mu}_4} F_{\breve{\mu}_5 \cdots \breve{\mu}_8} A_{\breve{\mu}_9 \breve{\mu}_{10}}], \end{aligned} \tag{53}$$

where $e^{(10)} \equiv \det e^{(10)\,\breve{\alpha}}_{\breve{\mu}}$, $\mathcal{R}$ is the Ricci scalar defined in terms of the Zehnbein, $F_{\breve{\mu}\breve{\nu}\breve{\rho}} \equiv 3\partial_{[\breve{\mu}} A_{\breve{\nu}\breve{\rho}]}$, $G_{\breve{\mu}\breve{\nu}} \equiv 2\partial_{[\breve{\mu}} B_{\breve{\nu}]}$, $F'_{\breve{\mu}\breve{\nu}\breve{\rho}\breve{\sigma}} \equiv 4\partial_{[\breve{\mu}} A_{\breve{\nu}\breve{\rho}\breve{\sigma}]} - 4F_{[\breve{\mu}\breve{\nu}\breve{\rho}} B_{\breve{\sigma}]}$, and $\varepsilon^{\breve{\mu}_1 \cdots \breve{\mu}_{10}} \equiv \varepsilon^{\breve{\mu}_1 \cdots \breve{\mu}_{10} 11}$. The ferminoic sector in 10-d contains Majorana gravitino $\psi_{\breve{\mu}}$ and fermion ψ_{11} that come from the 11-d gravitino $\psi^{(11)}_M$, *i.e.*, $\psi^{(11)}_M = (\psi_{\breve{\mu}}, \psi_{11})$. These spinors can be split into two Majorana-Weyl spinors of left- and right-helicities.

In order to obtain the effective 4-d action of the type-IIA superstring compactified on a 6-torus, one chooses the following KK Ansatz for the Zehnbein:

$$e^{(10)\,\breve{\alpha}}_{\breve{\mu}} = \begin{pmatrix} e^{\alpha}_{\mu} & \bar{B}^m_{\mu} \bar{e}^a_m \\ 0 & \bar{e}^a_m \end{pmatrix}, \tag{54}$$

where $\bar{B}^m_{\mu}$ $(m = 1, ..., 6)$ are Abelian KK gauge fields, e^{α}_{μ} is the string frame 4-d Vierbein and $\bar{e}^a_m$ is the Sechsbein. In the following, we shall set all the other scalars, except those

associated with the Sechsbein $\bar{e}^a_m$ and the 10-d dilaton Φ, to zero [13]. In this case, the string-frame 4-d bosonic action for the type-IIA superstring is of the following form:

$$\begin{aligned}\mathcal{L}_{II} = & -\frac{1}{4}e[e^{-2\phi}(\mathcal{R}+4\partial_\mu\phi\partial^\mu\phi+\frac{1}{4}\partial_\mu\bar{g}_{mn}\partial^\mu\bar{g}^{mn}-\frac{1}{4}\bar{g}_{mn}\bar{G}^m_{\mu\nu}\bar{G}^{n\,\mu\nu}-\bar{g}^{mn}\bar{F}_{\mu\nu\,m}\bar{F}^{\mu\nu}{}_n) \\ & +\frac{1}{4}e^{\bar{\sigma}}\bar{G}_{\mu\nu}\bar{G}^{\mu\nu}+\frac{1}{2}e^{\bar{\sigma}}\bar{g}^{mn}\bar{g}^{pq}\bar{F}_{\mu\nu\,mp}\bar{F}^{\mu\nu}{}_{nq}],\end{aligned} \tag{55}$$

where $e \equiv \det e^\alpha_\mu$, $2\phi \equiv 2\Phi - \ln\det\bar{e}^a_m$ (parameterizing the *string coupling*), $\bar{\sigma} \equiv \ln\det\bar{e}^a_m$ (parameterizing the volume of 6-torus), $\bar{g}_{mn} \equiv \eta_{ab}\bar{e}^a_m\bar{e}^b_n = -\bar{e}^a_m\bar{e}^a_n$, and $\bar{G}^m_{\mu\nu} \equiv \partial_\mu\bar{B}^m_\nu - \partial_\nu\bar{B}^m_\mu$. Here, the field strengths $\bar{F}_{\mu\nu\,m}$, $\bar{G}_{\mu\nu}$ and $\bar{F}_{\mu\nu\,mn}$ are defined in terms of the Abelian gauge fields decomposed from 10-d two-form $A_{\breve{\mu}\breve{\nu}}$, one-form $B_{\breve{\mu}}$ and the three-form $A_{\breve{\mu}\breve{\nu}\breve{\rho}}$ fields, respectively. The following Einstein-frame action can be obtained by the Weyl rescaling $g_{\mu\nu} \to g^E_{\mu\nu} = e^{-2\phi}g_{\mu\nu}$:

$$\begin{aligned}\mathcal{L}_{II} = & -\frac{1}{4}e^E[\mathcal{R}^E-2\partial_\mu\phi\partial^\mu\phi+\frac{1}{4}\partial_\mu\bar{g}_{mn}\partial^\mu\bar{g}^{mn}-\frac{1}{4}e^{-2\phi}\bar{g}_{mn}\bar{G}^m_{\mu\nu}\bar{G}^{n\,\mu\nu}-e^{-2\phi}\bar{g}^{mn}\bar{F}_{\mu\nu\,m}\bar{F}^{\mu\nu}{}_n \\ & +\frac{1}{4}e^{\bar{\sigma}}\bar{G}_{\mu\nu}\bar{G}^{\mu\nu}+\frac{1}{2}e^{\bar{\sigma}}\bar{g}^{mn}\bar{g}^{pq}\bar{F}_{\mu\nu\,mp}\bar{F}^{\mu\nu}{}_{nq}],\end{aligned} \tag{56}$$

where $e^E \equiv \sqrt{-\det g^E_{\mu\nu}}$ and $\mathcal{R}^E$ is the Ricci scalar defined in terms of the Einstein-frame metric $g^E_{\mu\nu}$.

Since we have turned off the scalar fields associated with the 10-d $U(1)$ gauge field $B_{\breve{\mu}}$, thus the internal metric coefficients g_{m7} of 11-d SG, the $SO(7)$ symmetry among 7 KK gauge fields and among 21 3-form gauge fields, separately, breaks down to the $SO(6)$ symmetry, which *do not mix* the gauge fields of RR and NS-NS sectors. The RR sector consists of one KK gauge field $\bar{B}_\mu$, which transforms as a singlet of $SO(6)$, and fifteen 3-form $U(1)$ gauge fields $\bar{A}_{\mu\,mn}$, which transform as **15** antisymmetric representation of $SO(6)$. The NS-NS sector consists of six KK gauge fields $\bar{B}^m_\mu$ and six 3-form $U(1)$ gauge fields $\bar{A}_{\mu\,n}$, each of them transforming as a **6** vector representation of $SO(6)$. [14].

Given two classes (10) and (45) of supersymmetric solutions in 11-d SG, one can find the corresponding solutions in the type-IIA superstring which are associated with different Abelian gauge fields in (55). For this purpose, one has to relate the bosonic fields in 4-d type-IIA superstring action (56) to the ones in 4-d action (5) of compactified 11-d SG. This is done by keeping track of field decomposition and redefinition, and by comparing the compactification Ansätze in two different schemes, *i.e.*, one corresponding to 11-d → 10-d → 4-d and the other one corresponding to 11-d → 4-d. The expressions of fields in the 4-d

---

[13]We turn off the scalar fields B_m (m=4,...,9) associated with the 10-d $U(1)$ gauge field $B_{\breve{\mu}}$. These fields are related to the internal metric coefficients g_{m7} ($m = 1, \cdots, 6$) of 11-d SG.

[14]In order to have the full manifestation of the $SO(7)$ symmetry of 11-d SG in the BH solutions of type-IIA superstring, the scalar fields which are associated with the 10-d $U(1)$ gauge field $B_{\breve{\mu}}$ has to be included.

type-IIA superstring action (56) in terms of those in the 4-d action (5) of 11-d SG are given by

$$\phi = -\frac{3}{7}\varphi + \frac{1}{4}\ln\rho_{77}, \quad \bar{\sigma} = \frac{9}{7}\varphi + \ln\rho_{77}, \quad \bar{\rho}_{mn} = (\rho_{77})^{\frac{1}{6}}\rho_{mn},$$
$$\bar{B}^m_\mu = B^m_\mu, \quad \bar{B}_\mu = B^7_\mu, \quad \bar{A}_{\mu m} = A_{\mu m7}, \quad \bar{A}_{\mu\,mn} = A_{\mu\,mn}, \tag{57}$$

where $m, n = 1, ..., 6$ and $\bar{\rho}_{mn}$ is the unimodular part of the internal metric $\bar{g}_{mn}$ ($\bar{g}_{mn} = -e^{\bar{\sigma}/3}\bar{\rho}_{mn}$). Recall, the internal metric g_{mn} in 11-d SG is related to its unimodular part ρ_{mn} and φ as: $g_{mn} = -\mathrm{e}^{-2\varphi/7}\rho_{mn}$.

By using the relation (57), we shall obtain the corresponding dyonic BH solutions of type-IIA superstring and the dependence of the corresponding ADM mass on the asymptotic values of the string coupling e^{ϕ_∞}, the volume $e^{\bar{\sigma}_\infty}$ and the corresponding unimodular parts $\bar{\rho}_{mn\,\infty}$ of the internal metric of the 6-torus. We shall quote the ADM mass M_E in the Einstein frame, which is related to the one in the string frame as $M_s \equiv e^{-\phi_\infty}M_E$.

We classify the solutions according to the type of 11-d fields, *i.e.*, the Elfbein $E^{(11)\,A}_M$ and the 3-form $A^{(11)}_{MNP}$, from which the 4-d $U(1)$ gauge fields are originated. The first set of solutions is the one corresponding to the case of KK BH's. They fall into the following two sets:

- **Type-KNR solutions** Magnetic charge P associated with $\bar{B}_\mu$, the KK gauge field in the RR sector, and electric charge Q_m associated with $\bar{B}^m_\mu$, one of six KK gauge fields in the NS-NS sector:

$$e^{(\phi-\phi_\infty)} = \left(\frac{r - \mathbf{P}_\infty - \mathbf{Q}_{m\,\infty}}{r - \mathbf{P}_\infty}\right)^{\frac{1}{4}}, \quad e^{2(\bar{\sigma}-\bar{\sigma}_\infty)} = \frac{(r - \mathbf{P}_\infty - \mathbf{Q}_{m\,\infty})^2}{(r - \mathbf{P}_\infty)^{-1}(r - \mathbf{Q}_{m\,\infty})^3},$$
$$\bar{\rho}_{mm}/\bar{\rho}_{mm\,\infty} = \left(\frac{r - \mathbf{P}_\infty}{r - \mathbf{P}_\infty - \mathbf{Q}_{m\,\infty}}\right)^{\frac{5}{6}}, \quad \bar{\rho}_{kk}/\bar{\rho}_{kk\,\infty} = \left(\frac{r - \mathbf{P}_\infty - \mathbf{Q}_{m\,\infty}}{r - \mathbf{P}_\infty}\right)^{\frac{1}{6}} \quad (k \neq m),$$
$$M_E = |\mathbf{P}_\infty| + |\mathbf{Q}_{m\,\infty}| = e^{\bar{\sigma}_\infty/2}|P| + e^{-\phi_\infty+\bar{\sigma}_\infty/6}\bar{\rho}^{\frac{1}{2}}_{mm\,\infty}|Q_m|. \tag{58}$$

  The $SO(6)/SO(5)$ rotations on this solution induces $\frac{6\cdot5}{2} - \frac{5\cdot4}{2} = 5$ new magnetic charge degrees of freedom in the gauge fields $\bar{B}_m$. For the case electric charge Q and magnetic charge P_m are associated with B_μ and $\bar{B}^m_\mu$, respectively, one can obtain the corresponding solutions by imposing the electric-magnetic duality transformations.

- **Type-KNN solutions** Magnetic charge P_m associated with $\bar{B}^m_\mu$ and electric charge Q_n associated with $\bar{B}^n_\mu$, *i.e.*, both charges correspond to KK fields of NS-NS sector:

$$e^{(\phi-\phi_\infty)} = \left(\frac{r - \mathbf{Q}_{n\,\infty}}{r - \mathbf{P}_{m\,\infty}}\right)^{\frac{1}{4}}, \quad e^{2(\bar{\sigma}-\bar{\sigma}_\infty)} = \frac{r - \mathbf{P}_{m\,\infty}}{r - \mathbf{Q}_{n\,\infty}}, \quad \bar{\rho}_{mm}/\bar{\rho}_{m\,\infty} = \frac{r - \mathbf{P}_{m\,\infty} - \mathbf{Q}_{n\,\infty}}{(r - \mathbf{P}_{m\,\infty})^{\frac{1}{6}}(r - \mathbf{Q}_{n\,\infty})^{\frac{5}{6}}},$$
$$\bar{\rho}_{nn}/\bar{\rho}_{nn\,\infty} = \frac{(r - \mathbf{P}_{m\,\infty})^{\frac{5}{6}}(r - \mathbf{Q}_{n\,\infty})^{\frac{1}{6}}}{r - \mathbf{P}_{m\,\infty} - \mathbf{Q}_{n\,\infty}}, \quad \bar{\rho}_{\ell\ell}/\bar{\rho}_{\ell\ell\,\infty} = \left(\frac{r - \mathbf{Q}_{n\,\infty}}{r - \mathbf{P}_{m\,\infty}}\right)^{\frac{1}{6}} \quad (\ell \neq m, n),$$
$$M_E = |\mathbf{P}_{m\,\infty}| + |\mathbf{Q}_{n\,\infty}| = e^{-\phi_\infty+\bar{\sigma}_\infty/6}\bar{\rho}^{\frac{1}{2}}_{mm\,\infty}|P_m| + e^{-\phi_\infty+\bar{\sigma}_\infty/6}\bar{\rho}^{\frac{1}{2}}_{nn\,\infty}|Q_n|, \tag{59}$$

Upon imposing the $SO(6)/SO(4)$ rotations, one has the most general supersymmetric 6-d KK BH's with constraint $\sum P_i Q_i = 0$.

Secondly, we have the following classes of dyonic solutions that correspond to $U(1)$ gauge fields associated with the 11-d 3-form field $A^{(11)}_{MNP}$:

- **Type-HNR solutions** Magnetic charge P_m associated with $\bar{A}_{\mu\, m}$, one of six 3-form fields in the NS-NS sector, and the electric charge Q_{mn} associated with $\bar{A}_{\mu\, mn}$, one of fifteen 3-form fields in the RR sector:

$$
\begin{aligned}
e^{(\phi-\phi_\infty)} &= \left(\frac{r-2\mathbf{Q}_{mn\,\infty}}{r-2\mathbf{P}_{m\,\infty}-2\mathbf{Q}_{mn\,\infty}}\right)^{\frac{1}{4}}, \quad e^{2(\bar{\sigma}-\bar{\sigma}_\infty)} = \frac{(r-2\mathbf{P}_{m\,\infty})(r-2\mathbf{Q}_{mn\,\infty})}{(r-2\mathbf{P}_{m\,\infty}-2\mathbf{Q}_{mn\,\infty})^2}, \\
\bar{\rho}_{mm}/\bar{\rho}_{mm\,\infty} &= \frac{(r-2\mathbf{P}_{m\,\infty})^{-\frac{2}{3}}(r-2\mathbf{Q}_{mn\,\infty})^{\frac{5}{6}}}{(r-2\mathbf{P}_{m\,\infty}-2\mathbf{Q}_{mn\,\infty})^{\frac{1}{6}}}, \quad \bar{\rho}_{nn}/\bar{\rho}_{n\,\infty} = \frac{(r-2\mathbf{P}_{m\,\infty}-2\mathbf{Q}_{mn\,\infty})^{\frac{5}{6}}}{(r-2\mathbf{P}_{m\,\infty})^{\frac{2}{3}}(r-2\mathbf{Q}_{mn\,\infty})^{\frac{1}{6}}}, \\
\bar{\rho}_{\ell\ell}/\bar{\rho}_{\ell\ell\,\infty} &= \frac{(r-2\mathbf{P}_{m\,\infty})^{\frac{1}{3}}(r-2\mathbf{Q}_{mn\,\infty})^{-\frac{1}{6}}}{(r-2\mathbf{P}_{m\,\infty}-2\mathbf{Q}_{mn\,\infty})^{\frac{1}{6}}} \quad (\ell \neq m,n), \\
M_E &= 2|\mathbf{P}_{m\,\infty}| + 2|\mathbf{Q}_{mn\,\infty}| = 2e^{-\phi_\infty-\bar{\sigma}_\infty/6}\bar{\rho}^{-\frac{1}{2}}_{mm\,\infty}|P_m| + e^{\bar{\sigma}_\infty/6}\bar{\rho}^{-\frac{1}{2}}_{mm\,\infty}\bar{\rho}^{-\frac{1}{2}}_{nn\,\infty}|Q_{mn}|. \quad (60)
\end{aligned}
$$

The $SO(6)/SO(4)$ rotations induce $\frac{6\cdot 5}{2}-\frac{4\cdot 3}{2}=9$ new charge degrees of freedom. For the case of the electric charge $\bar{Q}_m$ coming from $\bar{A}_{\mu\, m}$ and magnetic charge P_{mn} coming from $A_{\mu\, mn}$, the corresponding solutions can be obtained by imposing the electric-magnetic duality transformations.

- **Type-HRR solutions** Magnetic charge P_{mn} coming from $A_{\mu\, mn}$ and electric charge Q_{mp} coming from $A_{\mu\, mp}$, both of which are the charges of $U(1)$ fields in R-R sector:

$$
\begin{aligned}
e^{(\phi-\phi_\infty)} &= 1, \quad e^{2(\bar{\sigma}-\bar{\sigma}_\infty)} = \frac{r-2\mathbf{P}_{mn\,\infty}}{r-2\mathbf{Q}_{mp\,\infty}}, \\
\bar{\rho}_{mm}/\bar{\rho}_{mm\,\infty} &= \left(\frac{r-2\mathbf{P}_{mn\,\infty}}{r-2\mathbf{Q}_{mp\,\infty}}\right)^{-\frac{2}{3}}, \quad \bar{\rho}_{nn}/\bar{\rho}_{nn\,\infty} = \frac{(r-2\mathbf{P}_{mn\,\infty})^{\frac{1}{3}}(r-2\mathbf{Q}_{mp\,\infty})^{\frac{2}{3}}}{r-2\mathbf{P}_{mn\,\infty}-2\mathbf{Q}_{mp\,\infty}}, \\
\bar{\rho}_{pp}/\bar{\rho}_{p\,\infty} &= \frac{r-2\mathbf{P}_{mn\,\infty}-2\mathbf{Q}_{mp\,\infty}}{(r-2\mathbf{P}_{mn\,\infty})^{\frac{2}{3}}(r-2\mathbf{Q}_{mp\,\infty})^{\frac{1}{3}}}, \quad \bar{\rho}_{\ell\ell} = \left(\frac{r-2\mathbf{P}_{mn\,\infty}}{r-2\mathbf{Q}_{mp\,\infty}}\right)^{\frac{1}{3}} \quad (\ell \neq m,n,p), \\
M_E &= 2|\mathbf{P}_{mn\,\infty}| + 2|\mathbf{Q}_{mp\,\infty}| = 2e^{\bar{\sigma}_\infty/6}\bar{\rho}^{-\frac{1}{2}}_{mm\,\infty}\bar{\rho}^{-\frac{1}{2}}_{nn\,\infty}|P_{mn}| + e^{\bar{\sigma}_\infty/6}\bar{\rho}^{-\frac{1}{2}}_{mm\,\infty}\bar{\rho}^{-\frac{1}{2}}_{p\,\infty}|Q_{mp}|, \quad (61)
\end{aligned}
$$

The $SO(6)/SO(2)$ transformations on this solution introduces $\frac{6\cdot 5}{2}-\frac{2\cdot 1}{2}=14$ charge degrees of freedom in the gauge fields $A_{\mu\, ij}$.

- **Type-HNN solutions** Magnetic charge P_m associated with $\bar{A}_{\mu\, m}$ and electric charge Q_n associated with $\bar{A}_{\mu\, n}$, *i.e.*, both charges arise from 3-form fields in the NS-NS sector

[15]:

$$
\begin{aligned}
e^{(\phi-\phi_\infty)} &= \left(\frac{r-2\mathbf{Q}_{n\,\infty}}{r-2\mathbf{P}_{m\,\infty}}\right)^{\frac{1}{4}}, \quad e^{2(\bar\sigma-\bar\sigma_\infty)} = \frac{r-2\mathbf{Q}_{n\,\infty}}{r-2\mathbf{P}_{m\,\infty}}, \\
\bar\rho_{mm}/\bar\rho_{mm\,\infty} &= \frac{(r-2\mathbf{P}_{m\,\infty})^{\frac{1}{6}}(r-2\mathbf{Q}_{n\,\infty})^{\frac{5}{6}}}{r-2\mathbf{P}_{m\,\infty}-2\mathbf{Q}_{n\,\infty}}, \quad \bar\rho_{nn}/\bar\rho_{nn\,\infty} = \frac{r-2\mathbf{P}_{m\,\infty}-2\mathbf{Q}_{n\,\infty}}{(r-2\mathbf{P}_{m\,\infty})^{\frac{5}{6}}(r-2\mathbf{Q}_{n\,\infty})^{\frac{1}{6}}}, \\
\bar\rho_{\ell\ell}/\bar\rho_{\ell\ell\,\infty} &= \left(\frac{r-2\mathbf{P}_{m\,\infty}}{r-2\mathbf{Q}_{n\,\infty}}\right)^{\frac{1}{6}} \quad (\ell \neq m, n), \\
M_E &= 2|\mathbf{P}_{m\,\infty}| + 2|\mathbf{Q}_{n\,\infty}| = e^{-\phi_\infty-\bar\sigma_\infty/6}\bar\rho_{mm\,\infty}^{-\frac{1}{2}}|P_m| + e^{-\phi_\infty-\bar\sigma_\infty/6}\bar\rho_{nn\,\infty}^{-\frac{1}{2}}|Q_n|. \quad (62)
\end{aligned}
$$

Upon imposing the $SO(6)/SO(4)$ rotations, one obtains the most general supersymmetric 4-d 2-form BH solutions with the constraint $\sum Q_i P_i = 0$.

In the expressions for the Einstein frame ADM mass M_E [string frame ADM mass $M_s = e^{-\phi_\infty} M_E$] the screened charges from the RR sector do not scale [scale as $e^{-\phi_\infty}$] with respect to the asymptotic string coupling e^{ϕ_∞}, while the screened charges from the NS-NS sector scale as $e^{-\phi_\infty}$ [scale as $e^{-2\phi_\infty}$], in agreement with a general analysis in Ref. [1]. Note also the following scaling dependence of screened charges on the asymptotic volume of the 6-torus, parameterized by $e^{\bar\sigma_\infty}$: charges, associated with the KK fields in the RR and NS-NS sector, scale as $e^{\bar\sigma_\infty/6}$ and $e^{\bar\sigma_\infty/2}$, respectively, while charges associated with the 3-form fields all scale as $e^{-\bar\sigma_\infty/6}$.

We would also like to comment on the symmetry between the solutions with gauge fields originating from the NS-NS sector, *i.e.*, KK gauge fields $\bar B^m_\mu$ and the $U(1)$ gauge fields $\bar A_{\mu\,m}$ originated from the 10-d 2-form $A_{\breve\mu\breve\nu}$. That is to say, NS-NS sector of the zero-slope limit of type-IIA string compactified on a 6-trous, which is the same as the zero-slope limit of the corresponding heterotic string with gauge fields in the left-moving sector turned off, has the $SO(6,6)$ target space symmetry. This symmetry transformations transform the six KK gauge fields $\bar B^m_\mu$ and the six 2-form $U(1)$ gauge fields $\bar A_{\mu\,m}$, along with the internal metric $\bar g_{mn}$ and the corresponding pseudo scalar fields $\bar A_{mn}$, while leaving the 4-d space-time metric and the 4-d dilaton ϕ intact. In fact, a discrete subset of $SO(6)$ transformations exchanges KK gauge fields and the 2-from $U(1)$ gauge fields in the NS-NS sector, and transforms the internal metric $\bar g_{mn}$ into its inverse $(\bar g^{-1})^{mn}$ [31], thus relating the Type-KNN and Type-HNN solutions. This discrete symmetry generalizes the duality between H-monopole solution and KK solution in 5-d superstring [21]. Ultimately, one would like to address the full U-duality symmetry E_7, which relates all the above solutions.

[15]Note, that for a special case with either $\bar P_m = 0$ or $\bar Q_n = 0$, the result reduces to a solution first found in Ref. [21], and it is related to H-monopoles of heterotic string [15,17].

V. CONCLUSIONS

We discussed two separate classes of 4-dimensional (4-d), supersymmetric, dyonic, spherically symmetric, black holes (BH's) in 11-d supergravity (SG) compactified on a 7-torus. The first class of solutions is associated with 7 $U(1)$ gauge fields B^i_μ ($i = 1, \cdots, 7$) of the Elfbein, *i.e.*, Kaluza Klein (KK) BH's, and the second class of solutions is associated with 21 $U(1)$ gauge fields $A_{\mu\, ij}$ ($i = 1, \cdots, 7,\ i < j$) originated from the 3-form field $A^{(11)}_{MNP}$, *i.e.*, 3-form BH's. Within each class, we found the supersymmetric solutions in the case where among scalar fields only those associated with the internal metric of a 7-torus are turned on.

The general supersymmetric KK BH's are obtained by imposing global SO(7) rotations on supersymmetric solutions with a diagonal internal metric, and with one magnetic P_j and one electic Q_k charges coming from different KK gauge fields. The general supersymmetric 3-form BH's are similarly obtained by imposing the SO(7) rotations on supersymmetric solutions with a diagonal internal metric, and with one magnetic P_{ij} and one electic Q_{ik} charges associated with the corresponding two 3-form $U(1)$ gauge fields. Both types of solutions, therefore, have constrained charge configurations.

We have also related the above solutions to the corresponding solutions in the RR and NS-NS sectors of type-IIA superstring compactified on a 6-torus. We related the ADM masses of these solutions to the screened charges from the gauge fields in the RR and NS-NS sectors. These screened charges scale with the asymptotic string coupling in accordance with the analysis is Refs. [1,10]. In addition, the scaling of the screened charges (of KK and 3-form $U(1)$ gauge fields in the RR and NS-NS sectors) with the asymptotic value of the volume of the 6-torus are also given.

The two classes of solutions, *i.e.*, the one arising from KK gauge fields and the one from 3-form $U(1)$ gauge fields have the same 4-d metric. Thus, we infer that there exists a larger T-duality (or U-duality) transformation ($\in E_7$), which does not affect the 4-d space-time of configurations, that relates the two classes of configurations. In particular, we found a discrete transformation which relates the two classes of supersymmetric solutions with a diagonal internal metric. Such a larger duality symmetry contains, as a subset, the $SO(6,6)$ target space duality symmetry of the NS-NS sector of type-IIA superstrings compactified on a 6-torus, whose discrete subset transforms KK and 2-form $U(1)$ gauge fields in the NS-NS sector into one another. Thus, this larger T-duality symmetry reduces to $SO(6,6)$ T-duality of type-IIA superstring with the RR sector turned off, or equivalently, to the $SO(6,6)$ T-duality symmetry of the heterotic superstring with gauge fields of the left-moving sector turned off.

The ultimate goal is to find the generating solution which, supplemented by a subset of E_7 transformations, would generate *all* the supersymmetric BH solutions with all the scalar fields turned on. Subsets of E_7 transformations on the generating solution with a particular choice of charge configurations would in turn yield various classes of solutions which are discussed in this paper. In that manner, one would arrive at a unified picture [9] of all the 4-d, supersymmetric, spherically symmetric BH's in (type-IIA and heterotic) string theory; different classes of BH's associated with different types of $U(1)$ gauge fields are just the generating solutions viewed in different reference frame of T-duality.

ACKNOWLEDGMENTS

The work is supported by U.S. DOE Grant No. DOE-EY-76-02-3071, and the NATO collaborative research grant CGR 940870.

REFERENCES

[1] E. Witten, preprint IASSNS-HEP-95-18 (1995), hep-th# 9503124.
[2] C.M. Hull and P.K.Townsend, Nucl. Phys. **B438**, (1995) 109.
[3] P. K. Townsend, DAMTP-R-95-2, hep-th# 9501068.
[4] A. Sen, preprint TIFR-TH-95-16, hep-th# 9504027.
[5] J. A. Harvey and A. Strominger, preprint EFI-95-16, hep-th# 9504047.
[6] I. Bars, preprint, USC-95/HEP-B2, hep-th# 9503228.
[7] S.J. Gates and V.G.J. Rodgers, preprint UMDEPP-95-76, hep-th# 9503237.
[8] E. Bergshoeff, C. Hull, T. Ortin, preprint UG-3/95 and QMW-PH-95-4, hep-th# 9504081.
[9] P.K.Townsend, preprint DAMTP-R-95-12, hep-th# 9504095.
[10] A. Strominger, preprint HEPTH-9504090 (1995), hep-th# 9504090.
[11] B. Greene, D. Morrison and A. Strominger, CLNS-95/1335, hep-th# 9504145.
[12] C. Vafa, HUTP-95-A014, hep-th# 950523.
[13] G.W. Gibbons and C.M. Hull, Phys. Lett. **109B** (1982) 190.
[14] G.W. Gibbons and M. J. Perry, Nucl. Phys. **B248** (1984) 629.
[15] R. Khuri, Phys. Lett. **259B** (1991) 261; Phys. Lett. **294B** (1992) 325; Nucl. Phys. **B387** (1992) 315.
[16] G.T. Horowitz and A. Strominger, Nucl. Phys. **B360** (1991) 197.
[17] J.A. Harvey and J. Liu, Phys. Lett. **B268** (1991) 40.
[18] R. Kallosh, A. Linde, T. Ortín, A. Peet and A. van Proeyen, Phys. Rev. **D46** (1992) 5278.
[19] A. Giveon, E. Rabinovici, A.A. Tseytlin, Nucl. Phys. **B409** (1993) 339.
[20] A. Sen, preprint TIFR-TH-94-47 (1994), hep-th# 9411187 and references therein.
[21] T. Banks, M. Dine, H. Dykstra and W. Fischler, Nucl. Phys. **B212** (1988) 45.
[22] J.P. Gauntlett, J.A. Harvey and J. Liu, Nucl. Phys **B409** (1993) 365.
[23] M. Cvetič and D. Youm, preprint UPR-650-T (1995), hep-th# 9503081.
[24] M. Cvetič and D. Youm, Nucl. Phys. **B438**, (1995) 182.
[25] M. Cvetič and D. Youm, preprint UPR-649-T (1995), hep-th# 9503082.
[26] E. Cremmer and B. Julia, Phys. Lett. **76B** (1978) 409.
[27] E. Cremmer and B. Julia, Nucl. Phys. **B159** (1979) 141.
[28] M. Cvetič and D. Youm, preprint UPR-645-T (1995), hep-th# 9502099.
[29] J. Park, preprint CALT-68-1983 (1995), hep-th# 9503084.
[30] M. Huq and M.A. Namazie, Class. Quant. Grav. **2** (1985) 293.
[31] M. Cvetič and D. Youm, work in progress.